

# Constraining axion coupling constants from measuring the Casimir interaction between polarized test bodies

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## Abstract

We propose an experiment for measuring the effective Casimir pressure between two parallel SiC plates with aligned nuclear spins. The prospective constraints on an axion-neutron coupling constant for both hadronic and GUT axions are calculated using the process of one-axion exchange. For this purpose, a general expression for the additional pressure arising between two polarized plates due to the exchange of one axion between their constituent fermions is derived. We demonstrate that only the polarization component perpendicular to the plates contribute to the pressure. The obtained pressure can be both repulsive and attractive depending on whether the polarizations of both plates are unidirectional or directed in opposite directions. It is shown that although the constraints on an axion-electron coupling obtained in the case of magnetized plates are not competitive, the constraints on an axion-neutron coupling found for plates with polarized nuclear spins are of the same order of magnitude of those obtained previously for the GUT axions alone using the process of two-axion exchange. The proposed experiment allows also to strengthen the presently known constraints on the axion-neutron coupling constants of GUT axions by using both processes of one- and two-axion exchange.

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## I. INTRODUCTION

Axions are light pseudo-scalar particles which were predicted long ago [1, 2], but up to the present have not been found experimentally, in spite of repeated attempts. Much attention given to axions on both theoretical and experimental sides is determined by the important role they play in elementary particle physics, gravitation and cosmology. In QCD, axions explain the absence of strong CP violation and of the large electric dipole moment of the neutron [3–5]. They also provide the most natural solution for the problem of dark matter [3, 6].

The originally introduced QCD axions [1, 2] are pseudo-Nambu-Goldstone bosons. They interact with fermions via a pseudo-vector Lagrangian. Rather quickly, the prediction of Refs. [1, 2] was constrained to a very narrow band in parameter space [7], and was generalized to “invisible” hadronic axions [8, 9] possessing much smaller interaction constants. Another type of the so-called grand unified theory (GUT) axions, or axion-like particles, was proposed in Refs. [10, 11]. In the context of many models, the GUT axions interact with fermions via a pseudo-scalar Lagrangian [3].

Experimental searches for axions and axion-like particles are numerous and diverse [4, 5, 7, 12]. They are based on the interaction of axions with photons, electrons and nucleons. Many experiments use the so-called helioscopes and haloscopes [13], designed to register axions generated in the Sun [14–17] and constituting the dark matter [3, 6, 7, 18, 19], respectively. Up to now, however, only more or less strong constraints on the parameters of axions and axion-like particles have been obtained. Specifically, rather strong constraints on the coupling constants of hadronic axions were found from astrophysics [20] (see Ref. [21] for various models of hadronic axions). In this context, one could mention the constraints imposed by the neutrino data of supernova SN 1987A [22], from stellar cooling [23, 24], from direct Chandra observation of the surface temperature of isolated neutron stars [25], among others. It should be remembered, however, that astrophysical data, such as the emission rate, suffer from various uncertainties connected with dense matter effects [23].

The model-independent constraints on the coupling constants of neutrons to both hadronic and GUT axions were obtained from the magnetometer measurements using spin-polarized K and  $^3\text{He}$  atoms [26]. In so doing, an exchange of one axion between two neutrons has been used (see also Ref. [27] for other constraints obtained from neutron physics). Note

that the effective interaction potential between two fermions due to an exchange of one axion is spin-dependent, which is common irrespective of whether the axion-fermion interaction is described by the pseudo-vector or pseudo-scalar Lagrangian [28–30]. The model-independent laboratory constraints on the coupling constants of GUT axion-like particles to nucleons were obtained also from Eötvös- and Cavendish-type experiments [30–32], and from measurements of the Casimir-Polder and Casimir forces [33–36] (see also Ref. [37], for a review). Note also that a recent Casimir-less experiment [38], where the Casimir force is compensated in the measurement results, leads [39–41] to stronger constraints in the same mass-range, which are again valid for only the GUT axions. In this case, the two-axion exchange between two nucleons has been employed. The reason is that the test bodies used in the Eötvös- and Cavendish-type experiments, and in measurements of the Casimir interaction are unpolarized. As a result, an additional interaction due to one-axion exchange averages to zero. The effective potential due to two-axion exchange between two fermions was derived [30, 42, 43] with the assumption of a pseudo-scalar interaction Lagrangian (we recall that the quantum field theory based on the pseudo-vector Lagrangian is nonrenormalizable). This is the reason why all the constraints of Refs. [30–36] are applicable to the GUT axions (axion-like particles), but not to the hadronic axions.

In this paper, we propose an experiment for measuring the effective Casimir pressure between two polarized conductive plates. The suggested experiment would operate in much the same way as the already performed ones with Au test bodies [44–47] and magnetic (Ni), but not magnetized, test bodies [48–50]. The experiments of Refs. [44–47] were performed by means of the micromechanical torsional oscillator consisting of a Au-coated plate suspended at two opposite points on the midplane by serpentine springs. Two independent electrodes located under the plate were used to measure the capacitance between the plate and electrodes and to induce oscillations in the plate at the resonant frequency of the oscillator. Above the oscillator, a large Au-coated sphere was mounted on the side of an optical fiber in high vacuum. The Casimir force acting between a sphere and a plate changes the resonant frequency of the oscillator, and the measured frequency shift is proportional to the gradient of the Casimir force between a sphere and a plate. In consequence of the proximity force approximation [51, 52], this frequency shift is recalculated into the Casimir pressure between two plane parallel plates (see Sec. II). The experiments of Refs. [48–50] were performed by means of the dynamic atomic force microscope. In this case, a Au-coated sphere was at-

tached to a cantilever and oscillated harmonically above a Au-coated plate in high vacuum. The shift of the resonant frequency of an oscillator under the influence of the Casimir force was again the immediately measured quantity. It was recalculated into the gradient of the Casimir force between a sphere and a plate or into the Casimir pressure between two Au plates (see the review [52] for all experimental details).

First, we calculate an additional pressure which arises due to the process of one-axion exchange between fermions belonging to different plates. In so doing, all possible directions of polarization of each plate are considered. We show that, depending on the direction of polarization, the additional pressure can be both repulsive and attractive. Then we apply the obtained results to the case of magnetic polarization and find constraints on an axion-electron coupling constant which could be derived from an experiment of this kind. It is anticipated that the magnitude of the additional pressure due to the one-axion exchange is smaller than the measurement error which is assumed to be the same as in the experiments of Refs. [45, 46, 49, 50]. The obtained constraints on an axion-electron coupling constant are much weaker than the ones obtained from other experiments. Thus, the Casimir experiment with magnetized test bodies is not competitive.

An experiment for measuring the Casimir pressure between two polarized plates with aligned nuclear spins (i.e., possessing nuclear polarization) is shown to be more promising. We calculate the constraints on an axion-neutron coupling constant, which could be obtained by using the process of one-axion exchange from an experiment with SiC plates and found them quite competitive in the region of axion masses from  $10^{-3}$  to 1 eV. These constraints will be equally valid for both the hadronic and GUT axions (the constraints of Refs. [32–35] obtained in this region of masses are applicable only to GUT axions). Note that almost 100% degree of polarization of  $^{29}\text{Si}$  nuclear spins in silicon carbide has been observed recently [53]. This makes the performance of the proposed experiment quite possible. We also calculate the constraints on the axion-neutron coupling constant of the GUT axions which could be obtained from measurements of the Casimir pressure between SiC plates with aligned nuclear spins using simultaneously the processes of one- and two-axion exchange.

The paper is organized as follows. In Sec. II, we calculate the pressure which arises between two polarized plates due to the one-axion exchange between fermionic particles of their materials. Section III contains constraints on the axion-electron coupling constant which could be obtained from measuring the Casimir pressure between two magnetized

plates. In Sec. IV, we propose an experiment for measuring the Casimir pressure between SiC plates with aligned nuclear spins and calculate respective constraints on the axion-neutron coupling constants of both hadronic and GUT axions. Section V contains our conclusions and discussion.

Throughout the paper we use the system of units where  $\hbar = c = 1$ .

## II. CALCULATION OF THE PRESSURE BETWEEN TWO POLARIZED PLATES DUE TO ONE-AXION EXCHANGE

We consider two parallel material plates with densities  $\rho_i$  ( $i = 1, 2$ ) and thicknesses  $D_i$  separated by a gap of width  $a$ . We assume that each plate contains a fraction of atoms  $\kappa_i$  polarized in some definite direction (which can be different for different plates). The polarization of atoms may originate from the spin polarization of either electrons or nucleons (see Secs. III and IV, respectively).

Let us suppose we have a fermion with spin  $\boldsymbol{\sigma}_1/2$  (electron or nucleon) at a point  $\mathbf{r}_1$  of the first plate and a fermion with spin  $\boldsymbol{\sigma}_2/2$  at a point  $\mathbf{r}_2$  of the second plate. For both the pseudo-vector and pseudo-scalar interactions of an axion with fermions, the effective potential due to a one-axion exchange between two fermions has the common form [28–30]

$$V(r_{12}) = \frac{g^2}{16\pi m^2} \left[ (\boldsymbol{\sigma}_1 \cdot \mathbf{n})(\boldsymbol{\sigma}_2 \cdot \mathbf{n}) \left( \frac{m_a^2}{r_{12}^2} + \frac{3m_a}{r_{12}^2} + \frac{3}{r_{12}^3} \right) - (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) \left( \frac{m_a}{r_{12}^2} + \frac{1}{r_{12}^3} \right) \right] e^{-m_a r_{12}}. \quad (1)$$

Here,  $g$  is the dimensionless interaction constant of an axion with either an electron or nucleons (a neutron or a proton),  $m$  is the electron or nucleon mass,  $r_{12} = |\mathbf{r}_1 - \mathbf{r}_2|$  and  $\mathbf{n} = (\mathbf{r}_1 - \mathbf{r}_2)/r_{12}$ .

We consider first one polarized atom of the first plate situated at the point  $(x_1, y_1, z_1)$  above the upper surface of the second plate, which coincides with the coordinate plane  $(x, y)$ . Let the spins of the polarized atoms of the second (lower) plate be directed in the positive direction of the  $z$  axis. The polarized atoms of the second plate have the coordinates

$(x_2, y_2, z_2)$ . Then, we have

$$\begin{aligned}
r_{12} &= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2} \\
&= \sqrt{\rho^2 + (z_1 - z_2)^2}, \\
n_x &= \frac{x_1 - x_2}{r_{12}} = \frac{\rho \cos \varphi}{r_{12}}, \quad n_z = \frac{z_1 - z_2}{r_{12}},
\end{aligned} \tag{2}$$

where we have introduced polar coordinates  $(\rho, \varphi)$  in the coordinate plane  $(x, y)$  with the origin at the point  $(x_1, y_1)$ .

In experiments for measuring the Casimir force the linear sizes of the plates are much greater than  $a$ , whereas the strongest constraints on the coupling constants of an axion are obtainable at the Compton wavelengths  $m_a^{-1} \sim a$  [33–37]. Because of this, it is possible to consider plates as infinitely large discs and treat a polarized atom at the point  $(x_1, y_1, z_1)$  as situated above its center. Then, the interaction potential between an atom and a second plate due to the one-axion exchange is proportional to the fraction  $\kappa_2$  of the polarized atoms of the second plate and takes the form

$$V_1(z_1) = \kappa_2 \frac{\rho_2}{m_H} \int_{-D_2}^0 dz_2 \int_0^{2\pi} d\varphi \int_0^\infty \rho d\rho V(\rho, z_1, z_2), \tag{3}$$

where  $m_H$  is the mass of an atom of hydrogen, so that  $\rho_i/m_H$  is the number of atoms in the unit volume of the plate  $i$  and  $V$  is defined in Eq. (1).

Now we assume that the polarization of an atom of the first plate is directed either along the  $z$  axis or in opposition to it. Then, one obtains

$$(\boldsymbol{\sigma}_i \cdot \mathbf{n}) = \sigma_i \frac{z_1 - z_2}{r_{12}}, \quad (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) = \sigma_1 \sigma_2 \tag{4}$$

or

$$\begin{aligned}
(\boldsymbol{\sigma}_1 \cdot \mathbf{n}) &= -\sigma_1 \frac{z_1 - z_2}{r_{12}}, \quad (\boldsymbol{\sigma}_2 \cdot \mathbf{n}) = \sigma_2 \frac{z_1 - z_2}{r_{12}}, \\
(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) &= -\sigma_1 \sigma_2,
\end{aligned} \tag{5}$$

respectively.

Substituting Eqs. (1), (4) and (5) in Eq. (3), and integrating with respect to  $\varphi$ , for the two polarization directions of the atom one obtains

$$\begin{aligned}
V_1(z_1) &= \pm \sigma_1 \sigma_2 \frac{\kappa_2 \rho_2}{m_H} \frac{g^2}{8m^2} \int_{-D_2}^0 dz_2 \int_0^\infty \rho d\rho e^{-m_a r_{12}} \\
&\times \left[ \frac{m_a^2 (z_1 - z_2)^2}{r_{12}^3} + \frac{3m_a (z_1 - z_2)^2}{r_{12}^4} + \frac{3(z_1 - z_2)^2}{r_{12}^5} - \frac{m_a}{r_{12}^2} - \frac{1}{r_{12}^3} \right],
\end{aligned} \tag{6}$$

where  $r_{12}$  is given in Eq. (2).

It is convenient to rewrite Eq. (6) in terms of the new variable  $u = m_a r_{12}$ , which varies from  $u_1 = m_a(z_1 - z_2)$  to  $\infty$ :

$$V_1(z_1) = \pm g^2 \frac{\sigma_1 \sigma_2 \kappa_2 \rho_2 m_a}{8m^2 m_H} \int_{-D_2}^0 dz_2 I(u_1), \quad (7)$$

where

$$I(u_1) \equiv \int_{u_1}^{\infty} du e^{-u} \left( -\frac{1}{u} + \frac{u_1^2 - 1}{u^2} + \frac{3u_1^2}{u^3} + \frac{3u_1^2}{u^4} \right). \quad (8)$$

All the integrals in Eq. (8) are simply calculated [54] with the result

$$I(u_1) = e^{-u_1} = e^{-m_a(z_1 - z_2)}. \quad (9)$$

Substituting Eq. (9) in Eq. (7), one arrives at

$$\begin{aligned} V_1(z_1) &= \pm g^2 \frac{\sigma_1 \sigma_2 \kappa_2 \rho_2 m_a}{8m^2 m_H} \int_{-D_2}^0 dz_2 e^{-m_a(z_1 - z_2)} \\ &= \pm g^2 \frac{\sigma_1 \sigma_2 \kappa_2 \rho_2}{8m^2 m_H} e^{-m_a z_1} (1 - e^{-m_a D_2}). \end{aligned} \quad (10)$$

Note that if an atom of the first plate would be polarized not perpendicular, but along the surface of the second plate (for instance, along the  $x$  axis), this results in

$$(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) = 0, \quad (\boldsymbol{\sigma}_1 \cdot \mathbf{n})(\boldsymbol{\sigma}_2 \cdot \mathbf{n}) = \sigma_1 \sigma_2 \frac{\rho \cos \varphi (z_1 - z_2)}{r_{12}^2}. \quad (11)$$

Then, both terms in Eq. (1) do not contribute to the potential (3) (the first one turns into zero after an integration in  $\varphi$ , and the second one is identically equal to zero). Thus, there is no atom-plate interaction due to the exchange of one axion in this case.

Now we consider some other possible cases which could lead to the interaction between a polarized atom of the first plate and the second plate. Let the spins of the polarized atoms of the second (lower) plate be directed along its surface (for instance, along the  $x$  axis). The nontrivial situations are when the spin of a polarized atom of the first plate is either parallel or antiparallel to it. In these cases, we have

$$(\boldsymbol{\sigma}_1 \cdot \mathbf{n})(\boldsymbol{\sigma}_2 \cdot \mathbf{n}) = \sigma_1 \sigma_2 \frac{\rho^2 \cos^2 \varphi}{r_{12}^2} \quad (12)$$

or

$$(\boldsymbol{\sigma}_1 \cdot \mathbf{n})(\boldsymbol{\sigma}_2 \cdot \mathbf{n}) = -\sigma_1 \sigma_2 \frac{\rho^2 \cos^2 \varphi}{r_{12}^2}, \quad (13)$$

respectively, and the same respective results, as in Eqs. (4) and (5), are valid for  $(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)$ .

Substituting Eqs. (1), (12) and (13) in Eq. (3), for the two directions of polarization of an atom belonging to the first plate we find

$$V_1(z_1) = \pm \sigma_1 \sigma_2 \frac{\kappa_2 \rho_2}{m_H} \frac{g^2}{16\pi m^2} \int_{-D_2}^0 dz_2 \int_0^{2\pi} d\varphi \int_0^\infty \rho d\rho e^{-m_a r_{12}} \\ \times \left[ \frac{m_a^2 \rho^2 \cos^2 \varphi}{r_{12}^3} + \frac{3m_a \rho^2 \cos^2 \varphi}{r_{12}^4} + \frac{3\rho^2 \cos^2 \varphi}{r_{12}^5} - \frac{m_a}{r_{12}^2} - \frac{1}{r_{12}^3} \right]. \quad (14)$$

Carrying out the integration in Eq. (14) with respect to  $\varphi$ , and taking into account the definition of  $r_{12}$  in Eq. (12), one obtains

$$V_1(z_1) = \pm \sigma_1 \sigma_2 \frac{\kappa_2 \rho_2}{m_H} \frac{g^2}{16m^2} \int_{-D_2}^0 dz_2 \int_0^\infty \frac{\rho d\rho}{r_{12}} e^{-m_a r_{12}} \\ \times \left[ m_a^2 + \frac{m_a}{r_{12}} + \frac{1 - m_a^2 (z_1 - z_2)^2}{r_{12}^2} - \frac{3m_a (z_1 - z_2)^2}{r_{12}^3} - \frac{3(z_1 - z_2)^2}{r_{12}^4} \right]. \quad (15)$$

Now we introduce in Eq. (15) the new integration variable  $u = m_a r_{12}$  [the same as in Eq. (7)] and obtain

$$V_1(z_1) = \pm g^2 \frac{\sigma_1 \sigma_2 \kappa_2 \rho_2 m_a}{16m^2 m_H} \int_{-D_2}^0 dz_2 \tilde{I}(u_1), \quad (16)$$

where

$$\tilde{I}(u_1) \equiv \int_{u_1}^\infty du e^{-u} \left( 1 + \frac{1}{u} + \frac{1 - u_1^2}{u^2} - \frac{3u_1^2}{u^3} - \frac{3u_1^2}{u^4} \right). \quad (17)$$

Calculation of all integrals here [54] leads to the result  $\tilde{I}(u_1) = 0$ , and, thus,  $V_1(z_1) = 0$ , as well. One can conclude that if the polarizations of an atom and a plate are parallel to the plate surface there is no atom-plate force due to the process of one-axon exchange.

One more case to consider is when atoms of the second plate are polarized along its surface (for instance, along the  $x$  axis), whereas the atom of the first plate is polarized in the perpendicular direction (along the  $z$  axis). In this case, we obtain the same results as in Eq. (11), leading to the zero interaction potential (3).

Thus, we have considered six different options and found that nonzero interaction potentials (10) of opposite sign arise when the atomic polarizations are perpendicular to the plate surface and directed either in one direction or in the opposite directions. If the direction



of an atomic polarization makes some arbitrary angle to the plate surface, only its component perpendicular to the surface contributes to the interaction potential due to a one-axion exchange.

From Eq. (10) it is easy to obtain the interaction energy per unit area of two parallel plates due to a one-axion exchange between the polarized atoms belonging to these plates. For this purpose, we integrate Eq. (10) over the volume of the first plate with the coefficient taking into account the number of polarized atoms per unit volume. The result is

$$\begin{aligned} E(a) &= \pm g^2 \frac{\sigma_1 \sigma_2}{8m^2} \frac{\kappa_1 \kappa_2 \rho_1 \rho_2}{m_H^2} (1 - e^{-m_a D_2}) \int_a^{D_1+a} dz_1 e^{-m_a z_1} \\ &= \pm g^2 \frac{\sigma_1 \sigma_2}{8m^2 m_a} \frac{\kappa_1 \kappa_2 \rho_1 \rho_2}{m_H^2} e^{-m_a a} (1 - e^{-m_a D_1}) (1 - e^{-m_a D_2}). \end{aligned} \quad (18)$$

The most precise experiments on measuring the Casimir interaction [44–50] exploit the configuration of a sphere above a plate rather than of two parallel plates. In so doing, the immediately measured quantity is not the force  $F_{sp}$  acting between a sphere and a plate, but the force gradient  $\partial F_{sp}/\partial a$ . Using the proximity force approximation, which is very accurate under the condition  $a \ll R$  for both the Casimir and Yukawa-type forces [51, 52, 55, 56], one obtains

$$\frac{\partial F_{sp}(a)}{\partial a} = 2\pi R \frac{\partial E(a)}{\partial a} = -2\pi R P(a). \quad (19)$$

This equation expresses the force gradient between a sphere and a plate via the pressure  $P$  in the configuration of two parallel plates. Then the results of experiments [44–50] can be interpreted as measurements of the effective Casimir pressure between two parallel plates.

From Eqs. (18) and (19), the additional pressure between two parallel plates due to a one-axion exchange is found to be

$$P^{(1)}(a) = \mp g^2 \frac{\sigma_1 \sigma_2}{8m^2} \frac{\kappa_1 \kappa_2 \rho_1 \rho_2}{m_H^2} e^{-m_a a} (1 - e^{-m_a D_1}) (1 - e^{-m_a D_2}). \quad (20)$$

This result is used in Secs. III and IV for the cases of magnetized plates and plates with aligned nuclear spins.

### III. CONSTRAINING AXION-ELECTRON COUPLING CONSTANT FROM CASIMIR EXPERIMENT WITH MAGNETIZED TEST BODIES

In the experiment of Refs. [49, 50], the gradient of the Casimir force has been measured between the surfaces of a sphere and a plate coated by sufficiently thick layers of magnetic

metal Ni. The measurements were performed by means of an atomic force microscope. The measurement data were found to be in excellent agreement with the theoretical predictions of the Lifshitz theory using the tabulated optical data of Ni [57] extrapolated to lower frequencies by means of the plasma model. The theoretical prediction using the Drude model for an extrapolation were excluded by the measurement data. Recently, these results have been conclusively confirmed in Ref. [58] using a configuration, where theoretical predictions computed by means of the plasma and Drude models differ by up to a factor of 1000 [59].

The test bodies used in Refs. [49, 50] are magnetic, but not magnetized. What is more, it was shown [49, 50] that even for fully magnetized test bodies the gradient of the magnetic force acting between them is much below the instrumental sensitivity. This is because the magnetization results in a spatially homogeneous magnetic force at the submicrometer separations. This force leads to very minor contributions to the measured force gradient. Thus, it is possible to repeat the experiment [49, 50] with no other changes, but with the magnetized Ni test bodies in perpendicular direction to their surfaces.

For the magnetized Ni films, the magnetic moment of each atom is determined by a single electron of mass  $m = m_e$ . Then, the interaction constant  $g$  in Eq. (20) has the meaning of an axion-electron coupling constant  $g_{ae}$ . Note that for free Ni atoms  $\sigma_1 = \sigma_2 = \sigma = 1$ , whereas for Ni atoms included in a crystal lattice (as in our case)  $\sigma = 0.6$  [60]. Taking into account also that  $\kappa_1 = \kappa_2 = 1$ , Eq. (20) takes the form

$$|P^{(1)}(a)| = \frac{1}{8} g_{ae}^2 \left( \frac{\sigma}{m_e} \right)^2 \left( \frac{\rho_{\text{Ni}}}{m_{\text{H}}} \right)^2 e^{-m_a a} (1 - e^{-m_a D_1}) (1 - e^{-m_a D_2}), \quad (21)$$

where  $\rho_1 = \rho_2 = \rho_{\text{Ni}} = 8.9 \times 10^3 \text{ kg/m}^3$ . The thicknesses of Ni films used in the experiment of Refs. [49, 50] are the following:  $D_1 = 250 \text{ nm}$  and  $D_2 = 210 \text{ nm}$ .

The constraints on  $g_{ae}$  could be obtained from the assumption that in the experiment for measuring the gradient of the Casimir force between magnetized Ni test bodies the experimental data are in agreement with the same theory, as in Refs. [49, 50]. This means that no any additional force is detected, i.e., the magnitude of the pressure (21), due to one-axion exchange, satisfies the condition

$$|P^{(1)}(a)| < \frac{\Delta^{\text{tot}} F'_C}{2\pi R} = \Delta^{\text{tot}} P_C, \quad (22)$$

where  $\Delta^{\text{tot}} F'_C$  and  $\Delta^{\text{tot}} P_C$  are the total experimental errors in the measurements of the gradient of the Casimir force and of the effective Casimir pressure, respectively. In Refs. [49, 50]  $\Delta^{\text{tot}} F'_C = 1.2 \mu\text{N/m}$  at all separations considered, which results in  $\Delta^{\text{tot}} P_C = 3.1 \text{ mPa}$ .

Numerical analysis of Eqs. (21) and (22) shows that the strongest constraints on the quantity  $g_{ae}^2/(4\pi)$  are obtainable in the region of axion masses from 0.6 to 2 eV. Thus, for  $m_a = 0.65, 0.8, 1, 1.3$ , and 2 eV,  $g_{ae}^2/(4\pi)$  should be smaller than  $1.0 \times 10^{-9}$ ,  $9.5 \times 10^{-10}$ ,  $9.2 \times 10^{-10}$ ,  $1.0 \times 10^{-9}$ , and  $1.6 \times 10^{-9}$ , respectively. For comparison, the constraints on  $g_{ae}$  obtained in Ref. [61] for solar axions produced by the Compton process and bremsstrahlung are given by  $m_a g_{ae} \leq 3.1 \times 10^{-7}$  eV. For the axion mass  $m_a = 1$  eV, this results in  $g_{ae}^2/(4\pi) \leq 7.6 \times 10^{-15}$ , which is a much stronger constraint than the one following from Eq. (22). We conclude that experiments for measuring the Casimir interaction between two magnetized Ni test bodies are not a promising method for obtaining stronger constraints on the axion-electron coupling constant.

#### IV. PROPOSED EXPERIMENT USING TEST BODIES WITH ALIGNED NUCLEAR SPINS

Another possibility for obtaining constraints on the coupling constants of an axion from the Casimir effect using the process of one-axion exchange is to employ test bodies with nuclear polarization. It is well known that spin polarization can be transferred from electrons to nuclei. For metals this effect was first demonstrated theoretically and experimentally in the classical papers [62, 63]. In succeeding years, many different techniques for the dynamic nuclear polarization have been suggested [64–66]. It has been made possible to achieve high degrees of nuclear polarization, even at room temperature. This allowed producing nuclear polarized targets for particle physics and many other important applications.

Here, we propose a measurement of the effective Casimir pressure between two parallel plates made of silicon carbide (SiC) with aligned nuclear spins of Si. On the one hand, it was recently shown [53] that optically pumped nuclear polarization of  $^{29}\text{Si}$  nuclear spins in SiC can achieve 99% at room temperature. On the other hand, SiC is a semiconductor which can be doped both  $n$ -type and  $p$ -type [67]. Recently, it was used as a plate material in measurements of the Casimir force [68].

### A. Constraints on axion-neutron coupling constant using the process of one-axion exchange

Here, we propose an experiment for measuring the effective Casimir pressure between two all-silicon-carbide plates with aligned nuclear spins. Note that the nuclear spin of  $^{29}\text{Si}$  is equal to  $1/2$  ( $\sigma_1 = \sigma_2 = 1$ ) due to the presence of one neutron with an uncompensated spin. The natural abundance of  $^{29}\text{Si}$  is 4.6832%. There are, however, nanotechnology methods for growing isotopically controlled bulk Si [69]. Because of this, below we obtain the prospective constraints on an axion-neutron coupling constant  $g_{an}$ , using various values of  $\kappa_1 = \kappa_2 = \kappa$ . Taking into account that the actual thicknesses of the plates  $D_1$  and  $D_2$  are not yet available, we put them equal to infinity, i.e., consider two semispaces. This assumption works well as long as the Compton wavelength of an axion is much less than  $D$ . It is easy to take into consideration the actual values of  $D_i$ , as well as the boundary effects arising due to the finiteness of the plate area [35].

Taking into account all the above considerations, Eq. (21) for the magnitude of effective pressure due to one-axion exchange can be written in the form

$$|P^{(1)}(a)| = \frac{1}{8} g_{an}^2 \left( \frac{\kappa \rho_{\text{SiC}}}{m_n m_H} \right)^2 e^{-m_a a}, \quad (23)$$

where the density of SiC is  $\rho_{\text{SiC}} = 3.21 \text{g/cm}^3$ .

An experiment for measuring the effective Casimir pressure between nuclear polarized SiC plates could be performed similarly to Refs. [44–46], using a micromechanical torsional oscillator (this technique leads to a more precise results than that using an atomic force microscope). The total experimental error in Refs. [45, 46] is separation-dependent. The strongest constraints on  $g_{an}$  are obtainable at the separation distance  $a = 300 \text{ nm}$ , where  $\Delta^{\text{tot}} P_C = 0.22 \text{ mPa}$  [45, 46]. Then, the constraints on the quantity  $g_{an}^2/(4\pi)$  were found numerically from Eqs. (22) and (23).

In Fig. 1, the obtained constraints are shown as functions of the axion mass by the four dashed lines computed from top to bottom for the fractions  $\kappa$  of  $^{29}\text{Si}$  equal to 0.046832 (the natural abundance), 0.1, 0.5, and 1.0, respectively. The regions of  $[m_a, g_{an}^2/(4\pi)]$  plane above each line are excluded by the experimental results of the proposed experiment, and the regions below each line are allowed. As is seen in Fig. 1, the constraints become stronger with increasing fraction of  $^{29}\text{Si}$  isotope atoms whose nuclear spins are aligned. The strongest

constraint shown by the bottom dashed line is  $g_{an}^2/(4\pi) \leq 4.43 \times 10^{-5}$ . It is valid for axions with mass  $m_a = 0.0126 \text{ eV}$ .

The constraints following from this experiment would be valid for both hadronic and GUT axions. For comparison purposes, the solid line 1 in Fig. 1 shows the constraints on an axion-neutron coupling constant obtained [26] by means of a magnetometer, using spin polarized K and  $^3\text{He}$  atoms. These constraints found in the wide region of axion masses from  $10^{-10}$  to  $6 \times 10^{-6} \text{ eV}$  are also valid for both the hadronic and GUT axions. Similarly to our constraints, they were derived using the effective potential (1). As can be seen in Fig. 1, the constraints obtainable from the proposed measurements of the Casimir interaction between the test bodies with aligned nuclear spin and the constraints obtained from the magnetometer measurements complement each other nicely.

In Fig. 1 we also plot the constraints valid for only the GUT axions obtained from the gravitational experiments of Cavendish type [31, 32] (the solid line 2) and experiments on measuring the Casimir pressure between unpolarized test bodies [35] (the solid line 3) using the process of two-axion exchange. As is seen in Fig. 1, the proposed experiment could strengthen the previously known constraints on  $g_{an}$  for GUT axions in the region of axion masses  $4.4 \text{ meV} < m_a < 10 \text{ eV}$  and to extend them to both hadronic and GUT axions.

## B. Constraints on the coupling constant of axion-like particles to neutrons using the processes on one- and two-axion exchange

Now we calculate the constraints on  $g_{an}$  obtainable from the proposed Casimir experiment with nuclear polarized plates if the processes on one- and two-axion exchange are taken into account. The inclusion of the two-axion exchange into consideration restricts the region of applicability of the obtained results to the case of GUT axions only (see Sec. I). Although the interaction potential due to two-axion exchange is weaker than for an exchange of one axion, all nucleons of the test bodies contribute to it, and not just a small fraction of them with aligned spins.

The effective potential due to two-axion exchange between two nucleons (either protons or neutrons) belonging to the first and second plates is given by [30, 42, 43]

$$V_{kl}(r_{12}) = -\frac{g_{ak}^2 g_{al}^2 m_a}{32\pi^3 m^2} \frac{K_1(2m_a r_{12})}{r_{12}^2}, \quad (24)$$

where  $g_{ak}$  and  $g_{al}$  are the coupling constants of an axion-like particle to a proton ( $k, l = p$ ) or a neutron ( $k, l = n$ ),  $m = (m_p + m_n)/2$  is the mean nucleon mass, and  $K_1(z)$  is the modified Bessel function of the second kind.

An integration of Eq. (24) over the volumes of both plates (semispaces) leads to the following effective pressure due to the process of two-axion exchange [35]:

$$P^{(2)}(a) = -\frac{C^2}{2m^2 m_H^2} \int_1^\infty du \frac{\sqrt{u^2 - 1}}{u^2} e^{-2m_a a u}. \quad (25)$$

Here, the coefficient  $C$  for the plates made of SiC is defined as

$$C = \rho_{\text{SiC}} \left( \frac{g_{ap}^2}{4\pi} \frac{Z}{\mu} + \frac{g_{an}^2}{4\pi} \frac{N}{\mu} \right), \quad (26)$$

where  $Z$  and  $N$  are the numbers of protons and neutrons in the molecule SiC and  $\mu = m_{\text{SiC}}/m_H$ , with  $m_{\text{SiC}}$  being the mass of the SiC molecule. The values of  $Z/\mu$  and  $N/\mu$  for the first 92 elements of the Periodic Table (as well as the algorithms for calculating these quantities for molecules) are contained in Ref. [70].

We note that Eq. (25) depends on two unknown interaction constants,  $g_{an}$  and  $g_{ap}$ . As shown in Refs. [33–36], the weakest constraints on  $g_{an}$  are obtained under the condition  $g_{ap} \ll g_{an}$ . Being conservative, we use this condition now, which leads from Eq. (26) to the result

$$C \approx \rho_{\text{SiC}} \frac{g_{an}^2}{4\pi} \frac{N}{\mu}, \quad (27)$$

where for SiC one finds  $N = 20.11987$  and  $\mu = 39.78539$  [70].

The constraints on the coupling constant  $g_{an}$  of GUT axions (axion-like particles) can now be obtained from the equation

$$|P^{(1)}(a)| + |P^{(2)}(a)| \leq \Delta^{\text{tot}} P_C(a), \quad (28)$$

where  $P^{(1)}$  and  $P^{(2)}$  are defined in Eqs. (23) and (25). In so doing, we assume that the first and second plates are polarized in the opposite directions, so that  $P^{(1)}$  and  $P^{(2)}$  are negative, which corresponds to an attraction. The strongest constraints on  $g_{an}$  are again obtainable at  $a = 300$  nm, where  $\Delta^{\text{tot}} P_C(a) = 0.22$  mPa [45, 46].

In Fig. 2 we plot the obtained prospective constraints on  $g_{an}$  representing them by the dashed lines as functions of the axion mass. The top dashed line is computed for the fraction of polarized atoms  $\kappa$  between 0 and 0.1. In this case, the computational results do

not depend on whether the plates are polarized or unpolarized. The reason is that the total axionic pressure is determined by the two-axion exchange. The intermediate and bottom dashed lines are computed for the cases  $\kappa = 0.5$  and 1, respectively. Here, the nuclear polarization contributes essentially to a result which is determined by the joint action of one- and two-axion exchange.

For comparison purposes, the solid line in Fig. 2 reproduces the constraints on  $g_{an} \gg g_{ap}$  obtained [35] from an experiment on measuring the effective Casimir pressure between Au plates, using the process of two-axion exchange. As can be seen in Fig. 2, the intermediate dashed line ( $\kappa = 0.5$ ) shows the constraints of almost the same strength. In this case, the advantage of using the processes of both one- and two-axion exchange is reduced to zero due to the lower density of SiC, as compared with Au. However, the bottom dashed line ( $\kappa = 1$ ) shows up to a factor of 3.7 stronger constraints in comparison with the solid line. Thus, the proposed experiment allows not only to extend the previously known constraints to the case of hadronic axions, but also to strengthen some results obtained previously for the GUT axions.

## V. CONCLUSIONS AND DISCUSSION

In the foregoing, we have found the additional pressure which arises between two polarized parallel plates due to the process of one-axion exchange between the constituent fermions. Only the components of the polarizations which are normal to the plates are shown to contribute to the pressure. If the polarizations are unidirectional, the pressure is repulsive. If the polarizations are directed in opposite directions, then the pressure has the same magnitude, but it is attractive.

The obtained results were applied to the cases of magnetized plates and plates with aligned nuclear spins. In the first case, the performance of the experiment for measuring the effective Casimir pressure between two magnetized plates would constrain the axion-electron coupling constant  $g_{ae}$ . We have calculated the constraints on  $g_{ae}$  obtainable in this way by using the parameters of similar experiment already performed and found them not enough competitive. In the second case, the measurement of the effective Casimir pressure between two plates possessing nuclear polarization would constrain the axion-nucleon coupling constants  $g_{an}$  and  $g_{ap}$ .

We have proposed an experiment for measuring the effective Casimir pressure between SiC plates which have already been successfully used in Casimir experiments [68]. This material was preferred because it allows almost 100% nuclear polarization of  $^{29}\text{Si}$  nuclei [53] due to the presence of one neutron with an uncompensated spin. The respective constraints on an axion-neutron coupling constant  $g_{an}$ , obtainable for both hadronic and GUT axions from the suggested experiment, were calculated for various fractions of  $^{29}\text{Si}$  using typical already obtained experimental parameters. These constraints are shown to be of the same strength, as obtained previously for GUT axions only using the process of two-axion exchange from experiments on measuring the effective Casimir pressure between two Au plates. The discussed constraints would be complementary to the constraints, obtained from the magnetometer measurements for axions of lower masses, which are also valid for both hadronic and GUT axions.

Finally, we have calculated the constraints on the axion-neutron coupling constant  $g_{an}$  of GUT axions following from the proposed experiment with SiC plates if both processes of one- and two-axion exchange are taken into account. It is shown that, under some conditions, constraints stronger than those found previously from the experiment with two Au plate are obtainable.

In Secs. III and IV we have considered two different materials (Ni and SiC) possessing the electron and nuclear polarizations, respectively. If some material possesses both kinds of polarizations, this does not reduce a sensitivity of the proposed experiment to the axion-neutron interaction. This conclusion remains valid even if the electron and nuclear polarizations cancel each other. The point is that interactions of axions with the magnetic moments of electrons and neutrons are quite independent. Furthermore, at the experimental separations below  $1\,\mu\text{m}$ , the magnetic field, if any, is space homogeneous and does not contribute to the measured force gradient. One can also consider not the case of nuclear spins polarized in the same directions (as we assumed above), but a periodic arrangement with zero net polarization in both plates. In this case a nonzero force due to one-axion exchange between nucleons can arise only under a condition that the periods in both plates are equal. Then, depending on a phase shift between the periodic structures in both plates, the resulting pressure due to one-axion exchange would vary between some  $-P_{\text{max}}$  and  $P_{\text{max}}$ . The value of  $P_{\text{max}}$  can be calculated with the help of Fourier expansions similar to Ref. [50]. The experimental procedure using the periodically arranged nuclear spins would demand



the lateral scanning of the first test body relative to the second one to find out whether or not the measured pressure depends on a phase shift.

To conclude, experiments for measuring the Casimir interaction between test bodies made of different materials remain to be prospective for constraining the predictions of fundamental physical theories. In the past, strong constraints on the Yukawa-type corrections to Newtonian gravity have been obtained in this way (see, e.g., Refs. [51, 71–73]). At the present time, measurements of the Casimir interaction are helpful for constraining the coupling constants of axions and axion-like particles. One might expect that new experiments for measuring the Casimir force will lead to further progress in both these directions.

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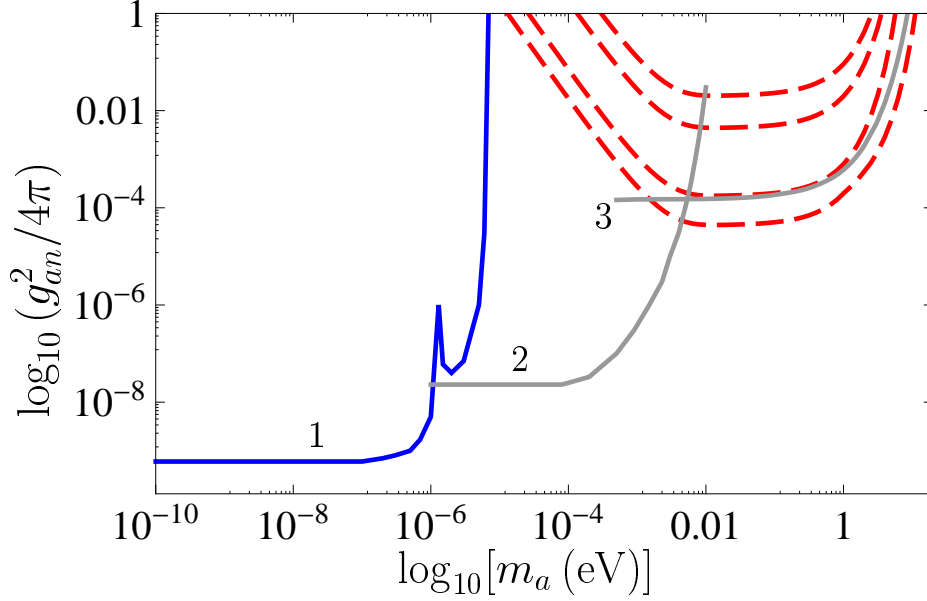


FIG. 1: (Color online) Constraints on the axion-neutron coupling constant of both hadronic and GUT axions obtained [26] from the magnetometer measurements (the solid line 1) and obtainable from the proposed experiment for measuring the effective Casimir pressure between SiC plates with aligned nuclear spins (the dashed lines from top to bottom correspond to the fractions of  $^{29}\text{Si}$  isotope equal to 0.046832, 0.1, 0.5, and 1.0, respectively) using the process of one-axion exchange are shown as functions of the axion mass. The solid lines 2 and 3 show the constraints for only the GUT axions found from the Cavendish-type experiment [31, 32] and from measurements of the Casimir pressure [35], respectively, using the process of two-axion exchange. The regions of the plane above each line are excluded and below each line they are allowed.

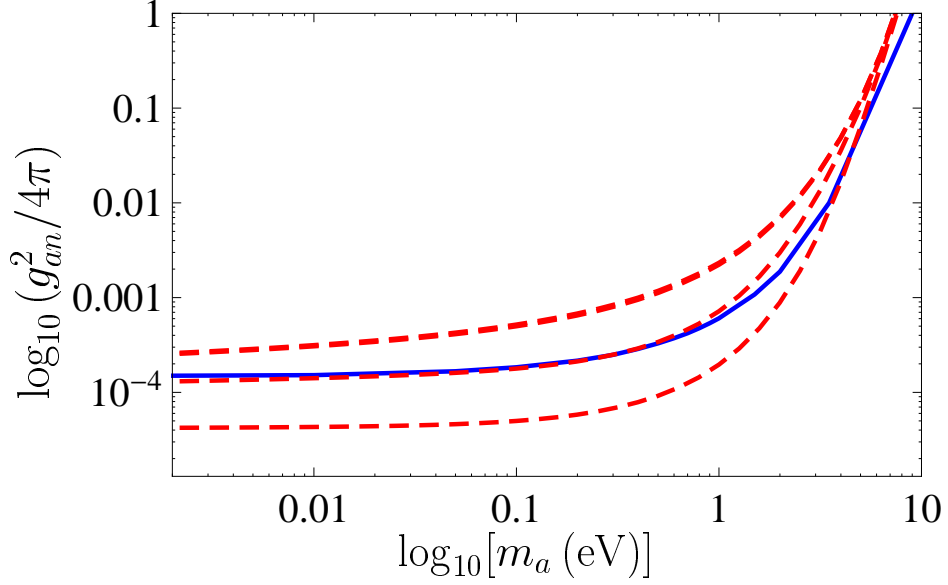


FIG. 2: (Color online) Constraints on the axion-neutron coupling constant of GUT axions already obtained [35] from measuring the effective Casimir pressure between two Au plates (the solid line) using the process of two-axion exchange and obtainable from similar proposed experiment using SiC plates with aligned nuclear spins (the dashed lines from top to bottom correspond to the fractions of  $^{29}\text{Si}$  isotope  $\leq 0.1$ ,  $0.5$ , and  $1.0$ , respectively) using the processes of one- and two-axion exchange are shown as functions of the axion mass. The regions of the plane above each line are excluded and below each line they are allowed.